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ON VARIANTS OF CM-TRIVIALITY

THOMAS BLOSSIER, AMADOR MARTIN-PIZARRO, AND FRANK O. WAGNER

ABSTRACT. We introduce a generalization of CM-triviality relative to a fixed invariant collection of partial types, in analogy to the Canonical Base Property defined by Pillay, Ziegler and Chatzidakis which generalizes one-basedness. We show that, under this condition, a stable field is internal to the family, and a group of finite Lascar rank has a normal nilpotent subgroup such that the quotient is almost internal to the family.

1. INTRODUCTION

Recall that a stable theory is called one-based if the canonical base of any real tuple \bar{a} over an algebraically closed set A is algebraic over \bar{a} . Hrushovski and Pillay have shown [9] that in a stable one-based group, definable sets are finite unions of cosets of $\text{acl}^{\text{eq}}(\emptyset)$ -definable subgroups. Furthermore, the group is abelian-by-finite. In the finite Lascar rank context, the notion of one-basedness agrees with local modularity and also with k -linearity (i.e. the canonical parameter of any uniformly definable family of curves has Lascar rank at most k) for any $k > 0$ (and in particular these notions agree for different k).

Hrushovski, in his *ab initio* strongly minimal set [10], introduced a weaker notion than one-basedness, CM-triviality, of which Baudisch's new uncountably categorical group [2] is a proper example. Pillay [15] showed that neither infinite fields nor bad groups could be interpretable in a CM-trivial stable theory; he concluded that a CM-trivial group of finite Morley rank must be nilpotent-by-finite. In [16] Pillay defined a whole hierarchy of geometrical properties (n -ampleness for $n > 0$), where one-based means non-1-ample, CM-trivial agrees with non-2-ample, and infinite fields are n -ample for all $n > 0$.

In [17], Pillay and Ziegler reproved the function field case of the Mordell-Lang conjecture in characteristic zero inspired by Hrushovski's original proof but avoiding the use of Zariski Geometries. Instead, motivated by work of Campana [6] and Fujiki [8], they showed in particular that the collection

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of types of finite Morley rank in the theory of differentially closed fields in characteristic zero has the *Canonical Base Property* (in short, CBP) with respect to the field of constants. That is, given a definable set X of bounded differential degree and Morley degree 1 in a saturated differentially closed field, the field of definition of the constructible set determined by X is almost internal to the constant field over a generic realisation of X . Furthermore, they showed the CBP for certain types in the theory of separably closed fields as well as for types of finite rank in the theory of generic difference fields of characteristic zero. The latter result was subsequently generalised by Chatzidakis [7] to all positive characteristics.

Clearly, the CBP is a generalisation of one-basedness, since for the latter the type of the canonical base of the generic type of X is already algebraic over a realisation. For the CBP, we replace algebraicity by almost internality with respect to a fixed invariant collection Σ of partial types. Chatzidakis [7] has shown that for a theory of finite rank, the CBP already implies a strengthening of this notion, called UCBP, introduced by Moosa and Pillay [13] in their study of compact complex spaces.

Kowalski and Pillay [12], generalizing the definability results obtained for one-based theories, showed that a connected group definable in a stable theory with the CBP is central-by-(almost Σ -internal).

The goal of this note is to introduce an analogous generalization of CM-triviality, again replacing algebraicity by almost Σ -internality. In view of the ampleness hierarchy, we call it 2-tightness (and 1-tightness is just the CBP). Similarly to the generalisation from one-based to the CBP, we show that a 2-tight stable field is Σ -internal, and a 2-tight group of finite Lascar rank is nilpotent-by-(almost Σ -internal). Recent work in [4] shows that the inclusion of the CM-trivial theories in the 2-tight ones is proper.

2. ONE-BASEDNESS AND 1-TIGHTNESS

The notions we will present could be generalised to any simple theory, replacing imaginary algebraic closure by bounded closure. However, the role of stability is essential for our main result, Theorem 3.6. We shall thus restrict ourselves to the stable case. In the following we will be working inside a sufficiently saturated model of a stable theory T . Note that we allow our elements and types to be imaginary, unless stated otherwise.

Definition 2.1. The theory T is *one-based* if for every real set A and every real tuple c the canonical base $\text{Cb}(c/A)$ is algebraic over c . Equivalently,

for every pair of real sets $A \subseteq B$ and every real tuple c , the canonical base $\text{Cb}(c/A)$ is algebraic over $\text{Cb}(c/B)$.

Remark 2.2. It was remarked in [9] that if T is one-based, the same conclusion holds for (possibly imaginary) sets A , B and tuples c . As noted in [15], we may assume that A (and B) are small submodels.

Recall some of the results proved by Hrushovski and Pillay [9]:

Fact 2.3. *Let G be an interpretable group in a one-based stable theory. Then:*

- (1) *Every connected definable subgroup H of G is definable over $\text{acl}^{\text{eq}}(\emptyset)$.*
- (2) *The connected component of G is abelian.*

Definition 2.4. The theory T is *CM-trivial* if for every pair of real sets $A \subseteq B$ and every real tuple c , if $\text{acl}^{\text{eq}}(Ac) \cap \text{acl}^{\text{eq}}(B) = \text{acl}^{\text{eq}}(A)$ then $\text{Cb}(c/A)$ is algebraic over $\text{Cb}(c/B)$.

Obviously, any one-based theory is CM-trivial.

Remark 2.5. As shown in [15], Remark 2.2 holds for CM-trivial theories as well.

Recall that a *bad group* is a connected group of finite Morley rank which is non-solvable and whose proper connected definable subgroups are all nilpotent. It is unknown whether bad groups exist. A bad group of minimal rank can be taken simple, i.e. with no proper normal subgroups, by dividing out by its finite centre.

Pillay shows in [15] the following:

Fact 2.6. *Let T be a CM-trivial stable theory. Then:*

- (1) *No infinite field can be interpreted.*
- (2) *An interpretable group of finite Morley rank is nilpotent-by-finite. In particular, no bad group can be interpreted.*

For the remainder of this article, we fix an \emptyset -invariant family Σ of partial types.

Definition 2.7. A type p over A is *internal to Σ* , or *Σ -internal*, if for every realisation a of p there is some superset $B \supset A$ with $a \downarrow_A B$, and realisations b_1, \dots, b_r of types in Σ based on B such that a is definable over B, b_1, \dots, b_r . If we replace definable by algebraic, then we say that p is *almost internal to Σ* or *almost Σ -internal*.

Poizat has shown that in a stable theory the parameter set B can be fixed independently of the choice of a .

Definition 2.8. We say that the theory T is *1-tight* with respect to Σ if for every tuple c and every set A , the type $\text{tp}(\text{Cb}(c/A)/c)$ is almost Σ -internal.

More generally, one can define 1-tight for a collection of partial types :

Definition 2.9. A collection \mathcal{F} of partial types in the theory T is *1-tight* with respect to Σ if for every set A_0 of parameters, every realisation c of a tuple of types in \mathcal{F} with parameters in A_0 and every $A \supseteq A_0$, the type $\text{tp}(\text{Cb}(c/A)/A_0c)$ is almost Σ -internal.

By definition, T is 1-tight if and only if the collection of all types over \emptyset is. Since we can assume that c contains the parameters A_0 , this is also equivalent to the collection of all types being 1-tight.

The *Canonical Base Property* CBP defined by Moosa, Pillay and Chatzidakis states that the collection of all types of finite Lascar rank is 1-tight with respect to the family of types of Lascar rank one. Our terminology provides for a possible generalisation of non-ampleness for higher values of n .

Remark 2.10. Clearly, a one-based theory is 1-tight with respect to any family Σ .

Lemma 2.11. *A theory T is 1-tight with respect to Σ if and only if for every pair of real sets $A \subset B$ and every real tuple c , the type $\text{tp}(\text{Cb}(c/A)/\text{Cb}(c/B))$ is almost Σ -internal.*

So Definition 2.8 generalises either of the two equivalent notions of one-basedness.

Proof. This is similar to [15, Remark 2.1]. Note that the requirement on A , B and \bar{c} to be real is no restriction, as we can replace imaginaries by sufficiently independent real representatives.

One implication is trivial by setting $B = \text{acl}^{\text{eq}}(Ac)$, since then $\text{Cb}(c/B)$ is interalgebraic with c . The other implication follows from the fact that

$$c \underset{\text{Cb}(c/B)}{\perp} B \text{ yields } c \underset{\text{Cb}(c/B)}{\perp} \text{Cb}(c/A)$$

and since a non-forking restriction of an almost internal type is again almost internal. \square

Adapting the proof of Fact 2.3, Kowalski and Pillay proved the following [12].

Fact 2.12. *Let G be a type-definable group in a stable theory which is 1-tight with respect to Σ .*

- (1) *Given a connected type-definable subgroup $H \leq G$, the type of its canonical parameter $\text{tp}(\ulcorner H \urcorner)$ is almost Σ -internal.*
- (2) *If G is connected, then $G/Z(G)$ is almost Σ -internal.*

Hrushovski, Palacín and Pillay [11] exhibit a theory of finite Morley rank without the CBP (i.e. which is not 1-tight with respect to the family of all types of Lascar rank one), elaborating an example of Hrushovski. However, if we replace almost internality by analysability, the resulting condition (strong Σ -basedness, or non weak 1- Σ -ampleness in the terminology of [14]) holds in any simple theory with enough regular types if we take Σ to be the family of all non one-based regular types [14, Theorem 5.1], generalising a result of Chatzidakis for types of finite SU-rank [7].

Question. Is every CM-trivial theory of finite rank 1-tight with respect to the family of all types of rank 1?

It is easy to see that if T is 1-tight with respect to Σ and Σ' results from Σ by removing all one-based types, then T is 1-tight with respect to Σ' .

3. DEFINABLE GROUPS IN 2-TIGHT THEORIES

In this section, we will introduce a weaker notion than CM-triviality, called 2-tightness, analogous to the generalisation from one-based to 1-tight. We will show that stable 2-tight fields and 2-tight simple groups of finite Lascar rank are Σ -internal. More generally, we show that a 2-tight group of finite Lascar rank is nilpotent-by-(almost Σ -internal).

Definition 3.1. The theory T is called *2-tight* with respect to Σ if for every pair of sets $A \subset B$ and every tuple c , if $\text{acl}^{\text{eq}}(Ac) \cap \text{acl}^{\text{eq}}(B) = \text{acl}^{\text{eq}}(A)$, then $\text{tp}(\text{Cb}(c/A)/\text{Cb}(c/B))$ is almost Σ -internal.

Again, one can also define 2-tight for a collection of partial types :

Definition 3.2. A collection \mathcal{F} of partial types in the theory T is *2-tight* with respect to Σ if for every set A_0 of parameters, every realisation c of a tuple of types in \mathcal{F} with parameters in A_0 and every $B \supseteq A \supseteq A_0$, if $\text{acl}^{\text{eq}}(Ac) \cap \text{acl}^{\text{eq}}(B) = \text{acl}^{\text{eq}}(A)$, then $\text{tp}(\text{Cb}(c/A)/A_0 \text{Cb}(c/B))$ is almost Σ -internal.

As before, T is 2-tight with respect to Σ if and only if the collection of all types is.

Remark 3.3. Every CM-trivial theory is 2-tight with respect to any family Σ . By Lemma 2.11, a 1-tight theory with respect to Σ is 2-tight with respect to Σ .

As in Remark 2.2, we may assume that A and B in the definition are small submodels. If T is 2-tight, the property also holds for imaginary tuples c .

Definition 3.4. A type p is *foreign* to Σ if every non-forking extension of p over B is orthogonal to every extension over B of any type in Σ .

We will repeatedly use the following connection between being foreign and internal for stable groups [19, Theorem 3.1.1 and Corollary 3.1.2]

Fact 3.5. (1) *If the generic type of a stable type-definable group G is not foreign to Σ , then there is a relatively definable normal subgroup N of infinite index such that G/N is Σ -internal.*
 (2) *A definably simple stable group or a stable division ring G whose generic type is not foreign to Σ must be Σ -internal. In particular, if its generic type is almost Σ -internal then G is Σ -internal.*

Inspired by Fact 2.6 we show the following.

Theorem 3.6. *Let T be stable and 2-tight with respect to Σ .*

- (1) *An interpretable field K is Σ -internal.*
- (2) *An interpretable group G of finite Lascar rank is nilpotent-by-(almost Σ -internal). In particular, an interpretable non-abelian simple group is Σ -internal.*

Proof. (1) We adapt the proof of [15, Proposition 3.2]. Let K be our interpretable field. Consider a generic plane P given by $ax + by + c = z$, where a, b, c are generic independent. Take a line ℓ contained in P given by $y = \lambda x + \mu$ such that λ, μ are generic independent over a, b, c . Take a point $p = (x, y, z)$ in $\ell \subset P$ generic over a, b, c, λ, μ .

Pillay has shown that $\text{acl}^{\text{eq}}(Ap) \cap B = A$, where $A = \text{acl}^{\text{eq}}(\ulcorner P \urcorner)$ is the imaginary algebraic closure of the canonical parameter of the plane and $B = \text{acl}^{\text{eq}}(\ulcorner P \urcorner \ulcorner \ell \urcorner)$ that of the parameters of the plane and the line. Furthermore, he has also shown that $\text{Cb}(p/A) = \ulcorner P \urcorner$ and $\text{Cb}(p/B) = \ulcorner \ell \urcorner$. By 2-tightness, $\text{tp}(\ulcorner P \urcorner / \ulcorner \ell \urcorner)$ is almost Σ -internal. Note that the line ℓ is given by the equations

$$\begin{aligned} y &= \lambda x + \mu \\ z &= (a + b\lambda)x + (c + b\mu). \end{aligned}$$

Since a, λ, μ, b, c are generic independent, so are $a, \lambda, \mu, b\lambda + a, c + b\mu$. Now

$$a \in \text{dcl}(\ulcorner P \urcorner) \quad \text{and} \quad \ulcorner \ell \urcorner \in \text{dcl}(a + b\lambda, c + b\mu, \lambda, \mu);$$

as $\text{tp}(\ulcorner P \urcorner / \ulcorner \ell \urcorner)$ is almost Σ -internal, so is $\text{tp}(a/a + b\lambda, c + b\mu, \lambda, \mu)$. Since

$$a \perp a + b\lambda, c + b\mu, \lambda, \mu,$$

we obtain that $\text{tp}(a)$ is almost Σ -internal. Thus K must be Σ -internal by Fact 3.5

(2) Let G be our group, which we may assume infinite. We treat first the case where G is simple non-abelian. We may assume that every type-definable connected subgroup of G is solvable: Otherwise we consider an infinite type-definable connected non-solvable subgroup G_0 of minimal Lascar rank, and a type-definable connected proper normal subgroup H_0 of G_0 of maximal rank. Then H_0 is solvable by minimality of rank, and contained in a relatively definable solvable proper normal subgroup H_1 of G_0 by stability. Moreover, every proper type-definable normal subgroup of G_0/H_1 is finite and hence central by connectedness, so $G_0/Z(G_0/H_1)$ is an infinite, type-definable, connected simple group of smaller Lascar rank than G . Inductively we may assume that this quotient is Σ -internal. But then the generic of G is not foreign to Σ by unidimensionality of simple groups of finite Lascar rank. Fact 3.5 yields that G is Σ -internal.

By superstability G contains an infinite type-definable solvable connected subgroup. If there is one, S , which is not nilpotent, then there is a section of G which interprets a field. By (1), this field is Σ -internal, and so is G by unidimensionality and Fact 3.5.

Otherwise, all proper connected type-definable subgroups of G are nilpotent. Let S be one of maximal Lascar rank and observe that S intersects trivially any other maximal type-definable connected nilpotent subgroup S_0 : Let $I = S \cap S_0$ be the intersection and assume S_0 is such that I is non-trivial of maximal rank possible. Since both S and S_0 are connected, the indices of I in S and in S_0 are both infinite. Hence so is the index of I in $N_S(I)$ because S is nilpotent [18, Proposition 1.12]. Simplicity of G yields that $N_G(I) \neq G$. Therefore, $N_G(I)^0$ is nilpotent and contained in some maximal type-definable connected nilpotent subgroup S_1 . But then $I < N_S(I)^0 \leq S_1 \cap S$ yields an intersection of larger rank, whence $S_1 = S$ by maximality. Now $I < N_{S_0}(I)^0 \leq S_1 \cap S_0 = S \cap S_0$ again provides an intersection of larger rank, a contradiction. In particular, S intersects all its distinct conjugates trivially. Moreover, as $N_G(S)^0$ must also be nilpotent,

maximality of S implies that S has bounded index in its normalizer. So

$$U\left(\bigcup_{g \in G} S^g\right) = U(G/N_G(S)) + U(S) = U(G/S) + U(S) = U(G)$$

and hence every generic element of G is contained in some conjugate of S .

We can now use the proof of [20, Lemma 2.5], which generalizes [15, Lemma 3.4 and Proposition 3.5], taking \mathfrak{B} to be the family of G -conjugates of S , and Q the family of all types of rank 1 (so Q -closure cl_Q equals imaginary algebraic closure). It is shown in [20] that for a, b, c in G generic independent and a generic conjugate S_0 of S over a, b, c , we have

$$\text{acl}^{\text{eq}}(b, c, A) \cap B = A,$$

where $A = \text{acl}^{\text{eq}}(\ulcorner(b, c) \cdot G_a \urcorner)$ is the imaginary algebraic closure of the canonical parameter of the coset determined by (b, c) of the subgroup

$$G_a = \{(g, g^a) \mid g \in G\},$$

and $B = \text{acl}^{\text{eq}}(\ulcorner P \urcorner \ulcorner l \urcorner)$, where l is the coset determined by (b, c) of the subgroup

$$S_a = \{(s, s^a) \mid s \in S_0\}.$$

Moreover, $\text{Cb}(b, c/A) = \ulcorner(b, c) \cdot G_a \urcorner$ and $\text{Cb}(b, c/B) = \ulcorner(b, c) \cdot S_a \urcorner$. By 2-tightness, the type $\text{tp}(\text{Cb}(b, c/A)/\text{Cb}(b, c/B))$ is almost Σ -internal.

Since G is simple non-abelian, its center is trivial, so a is definable over $\ulcorner G_a \urcorner$, which in turn is definable over $\ulcorner(b, c) \cdot G_a \urcorner = \text{Cb}(b, c/A)$. Furthermore, $\ulcorner(b, c) \cdot S_a \urcorner$ is definable over $\text{acl}^{\text{eq}}(b, c, \ulcorner S_0 \urcorner, \ulcorner a \cdot Z(S_0) \urcorner)$. Hence $\text{tp}(a/b, c, \ulcorner S_0 \urcorner, \ulcorner a \cdot Z(S_0) \urcorner)$ is almost Σ -internal. As b, c are generic independent from a and the parameters of S_0 , we have

$$\begin{array}{ccc} b, c & \downarrow & a, \\ & \ulcorner S_0 \urcorner, \ulcorner a \cdot Z(S_0) \urcorner & \end{array}$$

so the non-forking restriction $\text{tp}(a/\ulcorner S_0 \urcorner, \ulcorner a \cdot Z(S_0) \urcorner)$ is almost Σ -internal as well.

Now, since a is generic over $\ulcorner S_0 \urcorner$, it is generic in $aZ(S_0)$ over $\ulcorner a \cdot Z(S_0) \urcorner, \ulcorner S_0 \urcorner$ by stability. So the generic type of $aZ(S_0)$ is almost Σ -internal, and so is the generic type of $Z(S_0)$. Since S (and hence S_0) is nilpotent infinite, so is its center [18, Proposition 1.10]. Again unidimensionality of G and Fact 3.5 yield that G is Σ -internal.

Let now G be an arbitrary type-definable connected infinite group of finite Lascar rank in a 2-tight theory. By [3], there is a series of definable G -invariant subgroups

$$G = G_0 \triangleright G_1 \triangleright G_2 \triangleright \cdots \triangleright G_n = \{1\}$$

such that all quotients $Q_i = G_i/G_{i+1}$ for $i < n$ are either finite, abelian, or simple. In fact, applying repeatedly Fact 3.5 with respect to a single type of Lascar rank 1 which is either a completion of a type in Σ or foreign to Σ , we may refine the series and assume that every abelian quotient is unidimensional, and either Σ -internal or foreign to Σ . In the latter case, no definable section of the quotient is almost Σ -internal. Clearly all the finite quotients are Σ -internal; by the previous discussion, so are the simple quotients as well.

Let C be the centraliser in G of all Σ -internal quotients. If Q_i is abelian, then trivially $(C \cap G_i)' \leq C \cap G_{i+1}$; otherwise C centralises Q_i by definition, so again $(C \cap G_i)' \leq C \cap G_{i+1}$. Thus C is solvable.

Suppose that C^0 is not nilpotent. Then there is a definable section H of C^0 with a definable normal subgroup $K \triangleleft H$, such that K is the additive group of an infinite interpretable field and H/K is isomorphic to an infinite multiplicative subgroup $T \leq K^\times$, such that the multiplicative action of T on K^+ comes from the action of H/K on K by conjugation. By [18, Lemme 3.4] (or [19, Proposition 3.7.7] for the case of finite Lascar rank), the subgroup T generates K additively. It follows that K has no non-trivial H -invariant subgroup: If $K_0 \leq K$ is H -invariant and $0 \neq a \in K_0$, then $a^{-1}K_0$ is also H -invariant and contains T . As it is closed under addition, $a^{-1}K_0 \geq \langle T \rangle^+ = K$, whence $K_0 = K$. Note that K is Σ -internal by part (1).

Let \tilde{H} and \tilde{K} be the preimages of H and K in C , and let $i < n$ be minimal such that $\tilde{K} \leq G_i$. Since $\tilde{K} \cap G_{i+1}$ is normal in \tilde{H} , it induces an H -invariant subgroup of K , which must be trivial. Therefore K embeds definably into Q_i , which must be almost Σ -internal as K is. But then Q_i is centralised by C , so K is centralised by H . But this means that the multiplicative action of T on K^+ is trivial, a contradiction.

Finally, stability yields that $C_G(Q_i) = C_G(\bar{q}_i)$ for some finite tuple \bar{q}_i in Q_i . Hence any element $gC_G(Q_i)$ in $G/C_G(Q_i)$ is definable over the tuple $\bar{q}_i\bar{q}_i^g \in Q_i$. Therefore $G/C_G(Q_i)$ is Q_i -internal, and thus Σ -internal. Now G/C embeds definably into the product

$$\prod \{G/C_G(Q_i) : Q_i \text{ } \Sigma\text{-internal}\}$$

via the map

$$gC \mapsto (gC_G(Q_i) : Q_i \text{ } \Sigma\text{-internal}).$$

Therefore G/C is Σ -internal, and G/C^0 is almost Σ -internal. \square

Remark 3.7. If Σ contains all non one-based types of rank 1, [14, Theorem 6.6] implies in particular that a group G of finite Morley rank has a nilpotent subgroup N such that G/N is almost Σ -internal, without any hypothesis of 2-tightness. Hence part (2) above only provides additional information if Σ does not contain all non one-based types of rank 1. Note that if Σ contains all 2-ample types, then any simple theory is *weakly* 2-tight with respect to Σ by [14, Theorem 5.1], where we replace almost internality by analysability in the definition.

Question. What can one say about groups of infinite rank, or even merely stable ones? For instance, in a 2-tight stable group, can one find a nilpotent normal subgroup such that the quotient is almost Σ -internal?

Remark 3.8. The theory T_{FPS} of the free pseudospace [3] is not CM-trivial, and its collection of finite rank types is not 1-tight with respect to the family of rank 1 types (i.e. T_{FPS} does not have the CBP). However T_{FPS} is 2-tight with respect to the family of rank 1 types [4]. Unfortunately, T_{FPS} is trivial and therefore no infinite group is interpretable. Thus this example provides little insight into the significance of the above theorem.

Any almost strongly minimal theory is 1-tight and thus 2-tight with respect to the family of rank 1 types; an example of a 2-tight non 1-tight structure interpreting infinite groups is thus given by the disjoint union of the free pseudospace with an almost strongly minimal group. However, we don't know any example of a non-trivial 2-tight non 1-tight theory which does not decompose into a trivial and a 1-tight part.

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REFERENCES

- [1] A. Baudisch, *On superstable groups*, Trans. London Math. Soc. 42 (1990), 452–464.
- [2] A. Baudisch, *A new uncountably categorical group*, Trans. Amer. Math. Soc. 348 (1996), 3889–3940.
- [3] A. Baudisch and A. Pillay, *A free pseudospace*, J. Symb. Logic 65 (2000), 443–460.
- [4] A. Baudisch, A. Martin-Pizarro and M. Ziegler, *Ample Hierarchy*, in preparation.

- [5] C. Berline and D. Lascar, *Superstable groups*, Ann. Pure Appl. Logic 30 (1986), 1–43.
- [6] F. Campana, *Algébricité et compacité dans l'espace des cycles d'un espace analytique complexe*, Math. Ann. 251 (1980), 7–18.
- [7] Z. Chatzidakis, *A note on canonical bases and one-based types in super-simple theories*, Confluentes Mathematici, to appear. <http://arxiv.org/abs/1201.0134> (2011).
- [8] A. Fujiki, *On the Douady space of a compact complex space in the category C* , Nagoya Math. J. 85 (1982), 189–211.
- [9] E. Hrushovski and A. Pillay, *Weakly normal groups*, Stud. Logic Found. Math. 122 (1987), 233–244.
- [10] E. Hrushovski, *A new strongly minimal set*, Ann. Pure Appl. Logic 62 (1993), 147–166.
- [11] E. Hrushovski, D. Palacín and A. Pillay, *On the canonical base property*, preprint. <http://arxiv.org/abs/1205.5981> (2012).
- [12] P. Kowalski and A. Pillay, *Quantifier elimination for algebraic D -groups*, Trans. Amer. Math. Soc. 358 (2006), 167–181.
- [13] R. Moosa and A. Pillay, *On canonical bases and internality criteria*, Ill. J. Math. 52 (2008), 901–917.
- [14] D. Palacin and F.O. Wagner, *Ample thoughts*, J. Symb. Logic, to appear. <http://arxiv.org/abs/1104.0179> (2011).
- [15] A. Pillay, *The geometry of forking and groups of finite Morley rank*, J. Symb. Logic 60 (1995), 1251–1259.
- [16] A. Pillay, *A note on CM-triviality and the geometry of forking*, J. Symb. Logic 65 (2000), 474–480.
- [17] A. Pillay and M. Ziegler, *Jet spaces of varieties over differential and difference fields*, Selecta Math. 9 (2003), 579–599.
- [18] B. Poizat, *Groupes Stables, Une tentative de conciliation entre la géométrie algébrique et la logique mathématique. [An attempt at reconciling algebraic geometry and mathematical logic]*, Nur al-Mantiq wal-Ma'rifah, Lyon (1987).
- [19] F. O. Wagner, *Stable groups*, London Math. Soc. Lecture Note Series 240, Cambridge University Press, Cambridge (1997).
- [20] F. O. Wagner, *CM-triviality and stable groups*, J. Symb. Logic 63 (1998), 1473–1495.
- [21] B. Zilber, *Groups and rings whose theory is categorical*, (Russian), Fund. Math. 95 (1977), 173–188.

UNIVERSITÉ DE LYON; CNRS; UNIVERSITÉ LYON 1; INSTITUT CAMILLE JORDAN
UMR5208, 43 BOULEVARD DU 11 NOVEMBRE 1918, F-69622 VILLEURBANNE CEDEX,
FRANCE

E-mail address: `blossier@math.univ-lyon1.fr`

UNIVERSITÉ DE LYON; CNRS; UNIVERSITÉ LYON 1; INSTITUT CAMILLE JORDAN
UMR5208, 43 BOULEVARD DU 11 NOVEMBRE 1918, F-69622 VILLEURBANNE CEDEX,
FRANCE

E-mail address: `pizarro@math.univ-lyon1.fr`

UNIVERSITÉ DE LYON; CNRS; UNIVERSITÉ LYON 1; INSTITUT CAMILLE JORDAN
UMR5208, 43 BOULEVARD DU 11 NOVEMBRE 1918, F-69622 VILLEURBANNE CEDEX,
FRANCE

E-mail address: `wagner@math.univ-lyon1.fr`